

CIRCULANTS (Extract)

Alun Wyn-jones

Last revised in January 2008.

Please copy this book for your own reading only. Refers others to this website. Thank You.

Circulants.

| | |
|---|----|
| Chapter 1. Circulants | 1 |
| The Circulant Diagonalization Theorem | 3 |
| Circulant vectors. | 6 |
| Standard Bases for Circulant Space and Eigenspace. | 8 |
| The Representer Polynomial | 9 |
| The Circulant Determinant | 10 |
| Chapter 2. Circulant Matrices | 12 |
| The Centralizer of the Circulant Matrices. | 12 |
| The Shift-circulant or s -circulant Matrices | 12 |
| Normalizer of CIRC_N | 15 |
| The Linear Automorphisms of $\text{CIRC}_N(R_\zeta)$ | 18 |
| The Linear Automorphisms of $\text{CIRC}_N(\mathbb{R})$ | 19 |
| The Galois Group and Linear Automorphism of $\text{CIRC}_N(R)$ | 21 |
| Chapter 3. Homomorphisms | 22 |
| Introduction | 22 |
| δ -Idempotents | 22 |
| The Circulant Decomposition Theorem | 28 |
| The Polynomial Wrap-Around Map, Γ^N | 30 |
| Homomorphisms to Cyclotomic Fields | 31 |
| The Homomorphisms Γ_r^s | 33 |
| Restatement of the Circulant Decomposition Theorem | 35 |
| Group Rings | 39 |
| The Position Multiplier Maps | 41 |
| Chapter 4. The Supercirculants, \mathbf{circ}_∞ | 43 |
| Supercirculant Endomorphisms | 45 |
| Supercirculant Eigenvalues | 46 |
| The Inverse Transform, λ^{-1} | 47 |
| Chapter 5. Two Circulant Subalgebras | 49 |
| The Residue Class Matrices | 49 |
| Subrepeating Circulant Matrices | 52 |
| Decomposition of Subrepeating Circulants | 54 |
| Eigenvalue Decomposition of Subrepeating Circulants | 54 |
| Determinant Decomposition of Subrepeating Circulants | 55 |
| Chapter 6. Tensor Products | 58 |
| General Tensor Products of Circulant Matrices | 59 |
| Tensors of Supercirculants | 61 |
| Tensors Products and Polynomials in Several Variables | 62 |
| Chapter 7. Circulant Rings over the Integers and the Rationals | 64 |
| Introduction: The Group of Units in $\mathbf{circ}_N(\mathbb{Z})$ | 64 |
| Circulant and Cyclotomic Units of Finite Order | 68 |
| The Rational Circulants of Finite Order | 71 |
| Elements of Infinite Order in $\mathbf{U}(\mathbf{circ}_N(\mathbb{Z}))$ | 73 |
| A Theorem of G. Higman | 73 |
| Dirichlet's Unit Theorem | 74 |
| Kummer's Theorem and Cyclotomic Units | 74 |
| Bass Units and the Theorem of H. Bass | 75 |
| Fundamental Units for \mathcal{U}_p | 75 |
| Conclusions For The Prime Order Case | 81 |
| Prime Power Case | 81 |

| | |
|--|-----|
| Chapter 8. Irreducibles, Primes, and Ideals of $\mathbf{circ}_N(\mathbb{Z})$ | 86 |
| General Results | 86 |
| The Hilbert Basis Theorem | 87 |
| A Circulant Norm | 87 |
| Irreducibles | 88 |
| Primes | 90 |
| Cyclotomic Circulants and Cyclotomic Ideals | 90 |
| Factorizations | 94 |
| Non-unique Factorization in $\mathbf{circ}_p(\mathbb{Z})$ | 94 |
| Chapter 9. Application: Diffusion in Toroidal Spaces | 97 |
| Diffusion of Matter | 97 |
| Transitions Between States | 98 |
| Circulant Matrix Model | 98 |
| Boolean Circulants | 101 |
| Higher-dimensional Tori | 103 |
| Relaxation of the Assumptions | 104 |
| Chapter 10. Formulæ for the Circulant Determinant | 105 |
| Homogenous Diophantine Equations | 107 |
| Fermat’s Last Theorem | 108 |
| The Theorem of Sophie Germain | 109 |
| Wendt’s Circulant | 110 |
| Formulæ for the Determinantal Coefficients | 113 |
| Phase Formula | 114 |
| Parity Formula | 115 |
| Upper-Bounds on $ \Delta(a) $ | 117 |
| Chapter 11. Formula for Determinantal Coefficients | 120 |
| Notation of the Main Theorem | 120 |
| Statement of the Main Theorem | 121 |
| The Zero Set Formula | 121 |
| Ore’s Proof of the Zero Set Formula | 122 |
| Criticism of Ore’s Proof | 124 |
| New Proof of the Zero Set Formula | 125 |
| Zero Set Formula | 131 |
| Multiset Formula | 132 |
| Power Formula | 133 |
| Application to Permutations | 133 |
| Application to Cyclotomic Norms | 134 |
| Application to Combinatorics I | 135 |
| Application to Combinatorics II | 136 |
| The EGZ Theorem | 137 |
| Appendix A. Basic Cyclotomic Theory | 139 |
| Cyclotomic Extensions | 139 |
| Cyclotomic Polynomials | 139 |
| The Galois Group | 140 |
| Vector Space Basis | 140 |
| Cyclotomic Norm | 140 |
| Integral Elements | 141 |
| Appendix B. The Cooley-Tukey Fast Fourier Transform | 142 |
| Glossary of Terms | 145 |
| Notes and References | 148 |